

# Trion ladder diagrams

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## Abstract

We first derive a new “commutation technique” for an exciton interacting with electrons, inspired from the one we recently developed for excitons interacting with excitons. These techniques allow to take *exactly* into account the possible exchanges between carriers. We use it to get the  $X^-$  trion creation operator in terms of exciton and free-electron creation operators. In a last part we generate the ladder diagrams associated to these trions. Although similar to the exciton ladder diagrams, with the hole replaced by an exciton, they are actually much more tricky : (i) Due to the composite nature of the exciton, one cannot identify an exciton-electron potential similar to the Coulomb potential between electron and hole ; (ii) the spins are unimportant for excitons while they are crucial for trions, singlet and triplet states having different energies.

PACS number : 71.35.-y Excitons and related phenomena

The stability of semiconductor trions has been predicted long ago <sup>(1,2)</sup>. However, their binding energies being extremely small in bulk materials, clear experimental evidences <sup>(3,4)</sup> of these exciton-electron bound states have been achieved recently only, due to the development of good semiconductor quantum wells, the reduction of dimensionality enhancing all binding energies.

It is now possible to study these exciton-electron bound states as well as their interactions <sup>(5-7)</sup> with other carriers. However many-body effects with trions are even more subtle than many-body effects with excitons, due to their composite nature : Being made of indiscernable fermions, the interchange of these fermions with other carriers are quite tricky to handle properly.

We have recently developed a novel procedure to treat many-body effects between close-to-boson particles <sup>(8)</sup>, such as excitons in semiconductors. It allows to take *exactly* into account the possible exchanges between their components. In this paper, we first generate a similar procedure for an exciton interacting with electrons. We clearly see appearing an exciton-electron coupling induced by Coulomb interaction *and* an exciton-electron coupling induced by the possible exchange of the exciton electron with the other electrons. .

We use this commutation technique to get the trion creation operators in terms of exciton and free-electron creation operators and we show how, in the case of trions, all exchange couplings can be cleverly hidden in the prefactors of the trion operators, provided that these prefactors have a very specific invariance.

In a last part, we derive the trion ladder diagrams between a “free” exciton and a free electron. They are conceptually similar to the exciton ladder diagrams <sup>(9)</sup> between a free electron-hole pair, except for some quite specific difficulties associated to the composite nature of the exciton.

# 1 Commutation technique for an exciton interacting with electrons

The commutation technique for an exciton interacting with electrons relies on two parameters  $\Xi_{n'\mathbf{k}'n\mathbf{k}}^{\text{dir}}$  and  $\Lambda_{n'\mathbf{k}'n\mathbf{k}}$  which are such that

$$[V_{n,\sigma_n}^\dagger, a_{\mathbf{k},s}^\dagger] = \sum_{n',\mathbf{k}'} \Xi_{n'\mathbf{k}'n\mathbf{k}}^{\text{dir}} B_{n',\sigma_n}^\dagger a_{\mathbf{k}',s}^\dagger, \quad (1)$$

$$[D_{n',\sigma_{n'};n,\sigma_n}, a_{\mathbf{k},s}^\dagger] = \sum_{\mathbf{k}'} \delta_{\sigma_{n'},s} \Lambda_{n'\mathbf{k}'n\mathbf{k}} a_{\mathbf{k}',\sigma_n}^\dagger, \quad (2)$$

the “creation potential”  $V_{n,\sigma_n}^\dagger$  and the deviation-from-boson operator  $D_{n',\sigma_{n'};n,\sigma_n}$  being defined as for excitons interacting with excitons <sup>(8)</sup> by  $[H, B_{n,\sigma_n}^\dagger] = E_n B_{n,\sigma_n}^\dagger + V_{n,\sigma_n}^\dagger$  and  $[B_{n',\sigma_{n'}}, B_{n,\sigma_n}^\dagger] = \delta_{n,n'} \delta_{\sigma_n,\sigma_{n'}} - D_{n',\sigma_{n'};n,\sigma_n}$ .  $H$  is the semiconductor hamiltonian and  $B_{n,\sigma_n}^\dagger$  is the creation operator of the exciton  $n = (\nu_n, \mathbf{Q}_n)$ , with energy  $E_n = \epsilon_{\nu_n} + \hbar^2 \mathbf{Q}_n^2 / 2(m_e + m_h)$  and electron spin  $\sigma_n$ . (The hole “spin” playing no rôle here, we drop it to simplify the notations).  $B_{n,\sigma_n}^\dagger$  is linked to the electron and hole creation operators  $a_{\mathbf{k},s}^\dagger$  and  $b_{\mathbf{k}}^\dagger$  by

$$B_{n,\sigma_n}^\dagger = \sum_{\mathbf{p}} \langle \mathbf{p} | x_{\nu_n} \rangle b_{-\mathbf{p}+\alpha_h \mathbf{Q}_n}^\dagger a_{\mathbf{p}+\alpha_e \mathbf{Q}_n,\sigma_n}^\dagger, \quad (3)$$

$$b_{\mathbf{k}_h}^\dagger a_{\mathbf{k}_e,s}^\dagger = \sum_{\nu} \langle x_{\nu} | \alpha_h \mathbf{k}_e - \alpha_e \mathbf{k}_h \rangle B_{\nu,\mathbf{k}_e+\mathbf{k}_h,s}^\dagger, \quad (4)$$

where  $\alpha_{e,h} = m_{e,h} / (m_e + m_h)$ . By using the explicit expression of  $V_{n,\sigma_n}^\dagger$  deduced from its definition <sup>(8)</sup>, eq. (1) leads to

$$\Xi_{n'\mathbf{k}'n\mathbf{k}}^{\text{dir}} = \delta_{\mathbf{Q}_{n'}+\mathbf{k}',\mathbf{Q}_n+\mathbf{k}} W_{\nu_{n'}\nu_n}(\mathbf{Q}_{n'} - \mathbf{Q}_n), \quad (5)$$

$$W_{\nu'\nu}(\mathbf{q}) = V_{\mathbf{q}} \langle x_{\nu'} | e^{i\alpha_h \mathbf{q} \cdot \mathbf{r}} - e^{-i\alpha_e \mathbf{q} \cdot \mathbf{r}} | x_{\nu} \rangle, \quad (6)$$

$V_{\mathbf{q}}$  being the Fourier transform of the Coulomb potential. Let us mention that

$$\Xi_{n'\mathbf{k}'n\mathbf{k}}^{\text{dir}} = \int d\mathbf{r}_e d\mathbf{r}_{e'} d\mathbf{r}_h \phi_{n'}^*(\mathbf{r}_e, \mathbf{r}_h) f_{\mathbf{k}'}^*(\mathbf{r}_{e'}) \left[ \frac{e^2}{|\mathbf{r}_{e'} - \mathbf{r}_e|} - \frac{e^2}{|\mathbf{r}_{e'} - \mathbf{r}_h|} \right] \phi_n(\mathbf{r}_e, \mathbf{r}_h) f_{\mathbf{k}}(\mathbf{r}_{e'}), \quad (7)$$

where  $f_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} / \sqrt{\mathcal{V}}$  is the free-particle wave function while  $\phi_n(\mathbf{r}_e, \mathbf{r}_h) = \langle \mathbf{r}_{eh} | x_{\nu_n} \rangle f_{\mathbf{Q}_n}(\mathbf{R}_{eh})$ , with  $\mathbf{r}_{eh} = \mathbf{r}_e - \mathbf{r}_h$  and  $\mathbf{R}_{eh} = (m_e \mathbf{r}_e + m_h \mathbf{r}_h) / (m_e + m_h)$ , is the total wave function of the  $n$  exciton :  $\Xi_{n'\mathbf{k}'n\mathbf{k}}^{\text{dir}}$  thus corresponds to the scattering of a  $(n, \mathbf{k})$  state into a  $(n', \mathbf{k}')$  state induced by Coulomb interactions *between* the exciton and the electron, the  $n$  and  $n'$  excitons being made with the *same* electron-hole pair  $(e, h)$ .

If we turn to  $\Lambda_{n'\mathbf{k}'n\mathbf{k}}$ , eqs. (2,3) and the explicit expression of  $D_{n',\sigma';n,\sigma}$  deduced from its definition <sup>(8)</sup> lead to

$$\Lambda_{n'\mathbf{k}'n\mathbf{k}} = \delta_{\mathbf{Q}_{n'}+\mathbf{k}',\mathbf{Q}_n+\mathbf{k}} L_{\nu_{n'},\beta_x\mathbf{k}'-\beta_e\mathbf{Q}_{n'};\nu_n,\beta_x\mathbf{k}-\beta_e\mathbf{Q}_n} , \quad (8)$$

$$L_{\nu'\mathbf{p}'\nu\mathbf{p}} = \langle x_{\nu'} | \mathbf{p} + \alpha_e \mathbf{p}' \rangle \langle \mathbf{p}' + \alpha_e \mathbf{p} | x_{\nu} \rangle , \quad (9)$$

with  $\beta_e = 1 - \beta_x = m_e/(2m_e + m_h)$ . We can mention that

$$\Lambda_{n'\mathbf{k}'n\mathbf{k}} = \int d\mathbf{r}_e d\mathbf{r}_{e'} d\mathbf{r}_h \phi_{n'}^*(\mathbf{r}_e, \mathbf{r}_h) f_{\mathbf{k}'}^*(\mathbf{r}_{e'}) \phi_n(\mathbf{r}_{e'}, \mathbf{r}_h) f_{\mathbf{k}}(\mathbf{r}_e) , \quad (10)$$

which clearly shows that the  $(n, \mathbf{k})$  and  $(n', \mathbf{k}')$  states are coupled by  $\Lambda_{n'\mathbf{k}'n\mathbf{k}}$  due to their electron exchange independently from any Coulomb process. This possible exchange also leads to

$$B_{n,\sigma}^\dagger a_{\mathbf{k},s}^\dagger = - \sum_{n',\mathbf{k}'} \Lambda_{n'\mathbf{k}'n\mathbf{k}} B_{n',s}^\dagger a_{\mathbf{k}',\sigma}^\dagger , \quad (11)$$

while two exchanges reduce to identity :

$$\sum_{n'',\mathbf{k}''} \Lambda_{n'\mathbf{k}'n''\mathbf{k}''} \Lambda_{n''\mathbf{k}''n\mathbf{k}} = \delta_{nn'} \delta_{\mathbf{k}\mathbf{k}'} . \quad (12)$$

## 2 Trion creation operators

The  $X^-$  trions being made of two electrons and one hole, their creation operators write in terms of  $b_{\mathbf{k}_h}^\dagger a_{\mathbf{k}_e}^\dagger a_{\mathbf{k}_e'}^\dagger$ . According to eq. (4), they can also be written in terms of  $B_n^\dagger a_{\mathbf{k}}^\dagger$ , with  $\mathbf{Q}_n + \mathbf{k} = \mathbf{K}_i$ ,  $\mathbf{K}_i$  being the trion total momentum.

Let us consider the operators

$$T_{i,S,S_z}^\dagger = \sum_{\nu,\mathbf{p}} \tilde{\psi}_{\nu,\mathbf{p}}^{(\eta_i;S)} \mathcal{T}_{\nu,\mathbf{p},\mathbf{K}_i;S,S_z}^\dagger , \quad (13)$$

where  $i$  stands for  $(\eta_i, \mathbf{K}_i)$ , the four free exciton-electron creation operators  $\mathcal{T}_{\nu,\mathbf{p},\mathbf{K};S,S_z}^\dagger$  being given by

$$\begin{aligned} \mathcal{T}_{\nu,\mathbf{p},\mathbf{K};1,\pm 1}^\dagger &= B_{\nu,-\mathbf{p}+\beta_x\mathbf{K},\pm}^\dagger a_{\mathbf{p}+\beta_e\mathbf{K},\pm}^\dagger , \\ \mathcal{T}_{\nu,\mathbf{p},\mathbf{K};S,0}^\dagger &= \left( B_{\nu,-\mathbf{p}+\beta_x\mathbf{K},+}^\dagger a_{\mathbf{p}+\beta_e\mathbf{K},-}^\dagger - (-1)^S B_{\nu,-\mathbf{p}+\beta_x\mathbf{K},-}^\dagger a_{\mathbf{p}+\beta_e\mathbf{K},+}^\dagger \right) / \sqrt{2} . \end{aligned} \quad (14)$$

Due to eqs. (11,3,4,9), these operators are such that

$$\mathcal{T}_{\nu,\mathbf{p},\mathbf{K};S,S_z}^\dagger = (-1)^S \sum_{\nu',\mathbf{p}'} L_{\nu'\mathbf{p}'\nu\mathbf{p}} \mathcal{T}_{\nu',\mathbf{p}',\mathbf{K};S,S_z}^\dagger , \quad (15)$$

$$\langle v | \mathcal{T}_{\nu', \mathbf{p}', \mathbf{K}'; S', S'_z} \mathcal{T}_{\nu, \mathbf{p}, \mathbf{K}; S, S_z}^\dagger | v \rangle = \delta_{\mathbf{K}', \mathbf{K}} \delta_{S', S} \delta_{S'_z, S_z} \left( \delta_{\nu', \nu} \delta_{\mathbf{p}', \mathbf{p}} + (-1)^S L_{\nu' \mathbf{p}' \nu \mathbf{p}} \right) . \quad (16)$$

Equation (15) allows to replace the trion prefactors  $\tilde{\psi}$  in eq. (13) by  $\psi$  defined as

$$\psi_{\nu, \mathbf{p}}^{(\eta_i; S)} = \frac{1}{2} \left( \tilde{\psi}_{\nu, \mathbf{p}}^{(\eta_i; S)} + (-1)^S \sum_{\nu', \mathbf{p}'} L_{\nu \mathbf{p} \nu' \mathbf{p}'} \tilde{\psi}_{\nu', \mathbf{p}'}^{(\eta_i; S)} \right) , \quad (17)$$

so that, due to eq. (12), these prefactors now verify

$$\psi_{\nu, \mathbf{p}}^{(\eta_i; S)} = (-1)^S \sum_{\nu', \mathbf{p}'} L_{\nu \mathbf{p} \nu' \mathbf{p}'} \psi_{\nu', \mathbf{p}'}^{(\eta_i; S)} , \quad (18)$$

which just states that they stay invariant under the possible exchange corresponding to eq. (11). From eq. (13), with  $\tilde{\psi}$  replaced by  $\psi$ , and eqs. (16,18), one can easily check that

$$\langle v | \mathcal{T}_{\nu, \mathbf{p}, \mathbf{K}; S, S_z} \mathcal{T}_{\eta_i, \mathbf{K}_i; S_i, S_{iz}} | v \rangle = 2 \delta_{S, S_i} \delta_{S_z, S_{iz}} \delta_{\mathbf{K}, \mathbf{K}_i} \psi_{\nu, \mathbf{p}}^{(\eta_i; S)} . \quad (19)$$

$\mathcal{T}_{i, S, S_z}^\dagger | v \rangle$  is indeed a trion, i. e. an eigenstate of  $H$ , with the energy  $\mathcal{E}_{i, S}$ , if

$$\langle v | \mathcal{T}_{\nu, \mathbf{p}, \mathbf{K}_i; S, S_z} (H - \mathcal{E}_{i, S}) \mathcal{T}_{i, S, S_z}^\dagger | v \rangle = 0. \quad (20)$$

By using eqs. (14,15,19), we find that eq. (20) leads to

$$\left( \epsilon_\nu + \frac{\hbar^2 \mathbf{p}^2}{2\mu_t} - \varepsilon_{\eta_i; S} \right) \psi_{\nu, \mathbf{p}}^{(\eta_i; S)} + \sum_{\nu', \mathbf{p}'} W_{\nu \nu'}(-\mathbf{p} + \mathbf{p}') \psi_{\nu', \mathbf{p}'}^{(\eta_i; S)} , \quad (21)$$

where we have set  $\mathcal{E}_{i, S} = \varepsilon_{\eta_i; S} + \hbar^2 \mathbf{K}_i^2 / 2(2m_e + m_h)$ ,  $\mu_t$  being the exciton-electron relative motion mass,  $\mu_t^{-1} = m_e^{-1} + (m_e + m_h)^{-1}$ .

It can be surprising to note that the integral equation (21) verified by the trion prefactors  $\psi_{\nu, \mathbf{p}}^{(\eta_i; S)}$  only depends on the direct Coulomb scattering  $\Xi_{n\mathbf{K}n'\mathbf{K}'}^{\text{dir}}$ , through  $W_{\nu \nu'}(-\mathbf{p} + \mathbf{p}')$ . Exchange couplings  $\Lambda_{n\mathbf{K}n'\mathbf{K}'}$ , through  $L_{\nu \mathbf{p} \nu' \mathbf{p}'}$ , do not appear in it. They are actually hidden in the invariance relation (18) between the  $\psi_{\nu, \mathbf{p}}^{(\eta_i; S)}$ 's.

The trion orbital wave function deduced from eqs. (13,14) reads

$$\begin{aligned} F_{i, S}(\mathbf{r}_e, \mathbf{r}_{e'}, \mathbf{r}_h) &= f_{\mathbf{K}_i}(\mathbf{R}_{ee'h}) \sum_{\nu, \mathbf{p}} \psi_{\nu, \mathbf{p}}^{(\eta_i; S)} \left[ \langle \mathbf{r}_{eh} | x_\nu \rangle f_{\mathbf{p}}(\mathbf{u}_{e'eh}) + (-1)^S \langle \mathbf{r}_{e'h} | x_\nu \rangle f_{\mathbf{p}}(\mathbf{u}_{ee'h}) \right] / \sqrt{2} \\ &\equiv \sqrt{2} f_{\mathbf{K}_i}(\mathbf{R}_{ee'h}) \sum_{\nu, \mathbf{p}} \psi_{\nu, \mathbf{p}}^{(\eta_i; S)} \langle \mathbf{r}_{eh} | x_\nu \rangle f_{\mathbf{p}}(\mathbf{u}_{e'eh}) , \end{aligned} \quad (22)$$

with  $\mathbf{R}_{ee'h} = (m_e \mathbf{r}_e + m_{e'} \mathbf{r}_{e'} + m_h \mathbf{r}_h) / (2m_e + m_h)$  being the trion center of mass position, and  $\mathbf{u}_{e'eh} = \mathbf{r}_{e'} - \mathbf{R}_{eh}$  being the distance between the electron  $e'$  and the center of mass of the exciton made with  $(e, h)$ , the two terms of the first line of eq. (22) being equal due

to eqs. (18,9). As a consequence, the  $\psi$ 's can be obtained from the trion orbital wave function through

$$\sqrt{2}\psi_{\nu,\mathbf{p}}^{(\eta_i;S)} = \int d\mathbf{R} d\mathbf{r} d\mathbf{u} f_{\mathbf{K}_i}^*(\mathbf{R}) \langle x_\nu | \mathbf{r} \rangle f_{\mathbf{p}}^*(\mathbf{u}) F_{i,S}(\mathbf{R} + \alpha_h \mathbf{r} - \beta_e \mathbf{u}, \mathbf{R} + \beta_x \mathbf{u}, \mathbf{R} - \alpha_e \mathbf{r} - \beta_e \mathbf{u}) . \quad (23)$$

### 3 Trion ladder diagrams

It is known <sup>(9)</sup> that excitons correspond to “ladder diagrams” between one free electron and one free hole, which originate from the electron-hole Coulomb potential  $V$ . By writing the semiconductor hamiltonian as  $H = H_0 + V$ , these diagrams simply result from the iteration of  $(a - H)^{-1} = (a - H_0)^{-1} + (a - H)^{-1} V (a - H_0)^{-1}$  acting on one free electron-hole pair.

For trions, the problem appears at first as much more tricky, due to the composite nature of the exciton. There is indeed no way to write  $H$  as  $H'_0 + V'$ , with  $V'$  being an exciton-electron potential : This is in fact *the major difficulty* of all problems dealing with interacting excitons.

Our “commutation technique” allows to overcome this difficulty. Indeed, by using the equation which defines the “creation potential”  $V_{n,\sigma_n}^\dagger$ , we can show that

$$\frac{1}{a - H} B_{n,\sigma_n}^\dagger = B_{n,\sigma_n}^\dagger \frac{1}{a - H - E_n} + \frac{1}{a - H} V_{n,\sigma_n}^\dagger \frac{1}{a - H - E_n} . \quad (24)$$

For  $a = \Omega + i\eta$ , this equation, along with eqs. (1,5), gives  $(\Omega - H + i\eta)^{-1}$  acting on one exciton-free electron pair as

$$\begin{aligned} \frac{1}{\Omega - H + i\eta} \mathcal{T}_{\nu,\mathbf{p},\mathbf{K};S,S_z}^\dagger |v\rangle &= \frac{1}{\Omega - E_{\nu,\mathbf{p},\mathbf{K}} + i\eta} \left[ \mathcal{T}_{\nu,\mathbf{p},\mathbf{K};S,S_z}^\dagger |v\rangle \right. \\ &\quad \left. + \sum_{\nu',\mathbf{p}'} W_{\nu'\nu}(-\mathbf{p}' + \mathbf{p}) \frac{1}{\Omega - H + i\eta} \mathcal{T}_{\nu',\mathbf{p}',\mathbf{K};S,S_z}^\dagger |v\rangle \right] , \end{aligned} \quad (25)$$

with  $E_{\nu,\mathbf{p},\mathbf{K}} = \epsilon_\nu + \hbar^2 \mathbf{p}^2 / 2\mu_t + \hbar^2 \mathbf{K}^2 / 2(2m_e + m_h)$ .

The iteration of the above equation leads to

$$\frac{1}{\Omega - H + i\eta} \mathcal{T}_{\nu,\mathbf{p},\mathbf{K};S,S_z}^\dagger |v\rangle = \sum_{\nu',\mathbf{p}'} A_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K}) \mathcal{T}_{\nu',\mathbf{p}',\mathbf{K};S,S_z}^\dagger |v\rangle , \quad (26)$$

in which we have set

$$A_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K}) = \left[ \delta_{\nu',\nu} \delta_{\mathbf{p}',\mathbf{p}} + \frac{1}{\Omega - E_{\nu',\mathbf{p}',\mathbf{K}} + i\eta} (W_{\nu'\nu}(-\mathbf{p}' + \mathbf{p}) + \sum_{\nu_1, \mathbf{p}_1} \frac{W_{\nu'\nu_1}(-\mathbf{p}' + \mathbf{p}_1) W_{\nu_1\nu}(-\mathbf{p}_1 + \mathbf{p})}{\Omega - E_{\nu_1, \mathbf{p}_1, \mathbf{K}} + i\eta} + \dots) \right] \frac{1}{\Omega - E_{\nu, \mathbf{p}, \mathbf{K}} + i\eta}. \quad (27)$$

This  $A_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K})$  can be formally rewritten as

$$A_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K}) = \left[ \delta_{\nu',\nu} \delta_{\mathbf{p}',\mathbf{p}} + \frac{\tilde{W}_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K})}{\Omega - E_{\nu',\mathbf{p}',\mathbf{K}} + i\eta} \right] \frac{1}{\Omega - E_{\nu, \mathbf{p}, \mathbf{K}} + i\eta}, \quad (28)$$

where the “renormalized exciton-electron interaction”  $\tilde{W}_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K})$  verifies

$$\tilde{W}_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K}) = W_{\nu'\nu}(-\mathbf{p}' + \mathbf{p}) + \sum_{\nu_1, \mathbf{p}_1} \frac{\tilde{W}_{\nu'\mathbf{p}'\nu_1\mathbf{p}_1}(\Omega, \mathbf{K}) W_{\nu_1\nu}(-\mathbf{p}_1 + \mathbf{p})}{\Omega - E_{\nu_1, \mathbf{p}_1, \mathbf{K}} + i\eta}. \quad (29)$$

This iteration is shown in fig. (1). Before going further, let us note that

$$\frac{1}{\Omega - E_{\nu_1, \mathbf{p}_1, \mathbf{K}} + i\eta} \equiv \int \frac{id\omega_1}{2\pi} g_e(\Omega + \omega_1, \mathbf{p}_1 + \beta_e \mathbf{K}) g_x(-\omega_1; \nu_1, -\mathbf{p}_1 + \beta_x \mathbf{K}) = G_{xe}(\Omega; \nu_1, \mathbf{p}_1, \mathbf{K}), \quad (30)$$

where  $g_e(\omega, \mathbf{k}) = (\omega - \hbar^2 \mathbf{k}^2 / 2m_e + i\eta)^{-1}$  is the usual free electron Green’s function for an empty Fermi sea, while  $g_x(\omega; n) = (\omega - E_n + i\eta)^{-1}$  is the free boson-exciton Green’s function, as if the excitons were non-interacting bosons, i. e. if all the  $\Xi$ ’s and  $\Lambda$ ’s were set equal to zero.  $G_{xe}$  can be seen as the propagator of a free exciton-electron pair. It is quite similar to the free electron-hole pair propagator  $G_{eh}$  appearing in exciton diagrams (see eq. 2.10 of ref. (9)).

The simplest way to obtain  $A_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K})$  is to insert the trion closure relation between the two operators of the l.h.s. of eq. (26) and to project this equation over  $\langle v | \mathcal{T}_{\nu'', \mathbf{p}'', \mathbf{K}; S, S_z} \cdot$ . By using eqs. (19,16), we obtain

$$\sum_{\eta_i} \frac{4 \psi_{\nu'', \mathbf{p}''}^{(\eta_i; S)} \left( \psi_{\nu, \mathbf{p}}^{(\eta_i; S)} \right)^*}{\Omega - \mathcal{E}_{\eta_i, \mathbf{K}; S} + i\eta} = A_{\nu''\mathbf{p}''\nu\mathbf{p}}(\Omega, \mathbf{K}) + (-1)^S \sum_{\nu', \mathbf{p}'} L_{\nu''\mathbf{p}''\nu'\mathbf{p}'} A_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K}). \quad (31)$$

$A_{\nu''\mathbf{p}''\nu\mathbf{p}}(\Omega, \mathbf{K})$  is then obtained by adding the above equations for  $S = 0$  and  $S = 1$ . Using eq. (28), we thus get the sum of the exciton-electron ladder “rungs” as

$$\tilde{W}_{\nu'\mathbf{p}'\nu\mathbf{p}}(\Omega, \mathbf{K}) = \left[ -\delta_{\nu',\nu} \delta_{\mathbf{p}',\mathbf{p}} + \frac{1}{G_{xe}(\Omega; \nu', \mathbf{p}', \mathbf{K})} \sum_{\eta_i, S} \frac{2 \psi_{\nu', \mathbf{p}'}^{(\eta_i; S)} \left( \psi_{\nu, \mathbf{p}}^{(\eta_i; S)} \right)^*}{\Omega - \mathcal{E}_{\eta_i, \mathbf{K}; S} + i\eta} \right] \frac{1}{G_{xe}(\Omega; \nu, \mathbf{p}, \mathbf{K})}. \quad (32)$$

This result is quite similar to the “renormalized electron-hole Coulomb interaction” appearing in electron-hole ladder diagrams, as given for example in eq. (2.18) of ref. (9).

## 4 Conclusion

We have generated the exciton-electron ladder diagrams associated to trions. They will allow to treat trions in a similar way as excitons, with respect to many-body effects in which they can be involved. This work relies on a new commutation technique for excitons interacting with electrons which takes exactly into account the possible exchange between carriers.



## REFERENCES

- (1) E. HYLLERAAS, Phys. Rev. 71, 491 (1947).
- (2) M. LAMPERT, Phys. Rev. Lett. 1, 450 (1958).
- (3) G. FINKELSTEIN, H. SHTRIKMAN, I. BAR-JOSEPH, Phys. Rev. Lett. 74, 976 (1995).
- (4) A.J. SHIELDS, M. PEPPER, D.A. RITCHIE, M.Y. SIMMONS, G.A. JONES, Phys. Rev. B 51, 18049 (1995).
- (5) S.A. BROWN, J.F. YOUNG, J.A. BRUM, P. HAWRYLAK, Z. WASILEWSKI, Phys. Rev. B, Rapid Com. 54, 11082 (1996).
- (6) R. KAUR, A.J. SHIELDS, J.L. OSBORNE, M.Y. SIMMONS, D.A. RITCHIE, M. PEPPER, Phys. Stat. Sol. 178, 465 (2000).
- (7) V. HUARD, R.T. COX, K. SAMINADAYAR, A. ARNOULT, S. TATARENKO, Phys. Rev. Lett. 84, 187 (2000).
- (8) M. COMBESCOT, O. BETBEDER-MATIBET, Europhys. Lett. 58, 87 (2002) ; O. BETBEDER-MATIBET, M. COMBESCOT, Eur. Phys. J. B 27, 505 (2002).
- (9) O. BETBEDER-MATIBET, M. COMBESCOT, Eur. Phys. J. B 22, 17 (2001).

## FIGURE CAPTIONS

Fig (1)

- (a) Direct Coulomb scattering between one “free” exciton and one free electron.
- (b) Renormalized free exciton-electron interaction as given by the integral equation (29). It corresponds to the sums of one, two,  $\dots$  ladder “rungs” between one “free” exciton and one free electron.